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Paramagnetic state in d-wave Superconductors

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Abstract. – We study theoretically the paramagnetic state in d-wave superconductors. We present the specific heat, the magnetization, superfluid density obtained within the weak-coupling model. At low temperatures and for small magnetic fields they exhibit simple power law behaviors, which should be accessible experimentally in hole-doped high-T_c cuprates and κ-(ET)$_2$ salts in a magnetic field within the conducting plane.

Perhaps the most momentous event in the recent history of superconductivity is the identification of d-wave symmetry in the hole-doped high-T_c cuprates superconductors [1, 2]. We believe now most of newly discovered superconductors; heavy fermion supercondutors and organic superonductors are unconventional [3], though of course these possibilities have been widely discussed in the literature [4, 5].

First we would like to point out that the superconductivity in κ-(ET)$_2$ salts like the one in κ-(ET)$_2$Cu(N(CN)$_2$)Br is most likely of d_{xy}-wave [6]. Clearly earlier T_1 measurement of ^{13}C NMR [7], the specific data[8], and the more recent thermal conductivity data [9] as well as a recent measurement of the magnetic penetration depth in the superconducting state of κ-(ET)$_2$X [6] support that the superconductivity is of d_{xy}-wave.

As is well known both high-T_c cuprates and κ-(ET)$_2$salts have layered structure with weak interlayer coupling. One of the signatures of the weak interlayer coupling and d-wave superconductivity is the peak in the out-of-plane magnetoresistance just before the superconducting transition [3, 10] as seen by Ito et al. [11] and Kartsovnik et al. [12]. In these systems, we believe that the orbital effect is secondary when a magnetic field is applied parallel to the conducting plane. In such a situation the Pauli paramagnetism should be of prime importance [13]. In order to test this idea we study theoretically the paramagnetic state in d-wave superconductors. Perhaps κ-(ET)$_2$salts will provide an ideal system to test the theoretical prediction.

In this Letter we shall neglect deliberately the question of the nonhomogeneous superconducting state in d-wave superconductors [14, 15] analogous to the one discussed in s-wave

Fig. 1. – (a) The order parameter in d-wave superconductor is shown as a function of T/T_{c0} for several fields. (b) The order parameter in d-wave superconductor is shown as a function of h/Δ_{00} for several temperatures.

superconductors [16, 17], but rather concentrate on the paramagnetic state with uniform order parameter. Therefore our main purpose is to extend earlier works on s-wave superconductors [18, 19] to d-wave superconductors. Indeed the presence of nodes in d-wave order parameter gives rise to a number of new effects, some of which have been already discussed in [15]. In the following we take the weak-coupling model for d-wave superconductors [20] and study the paramagnetic state. First of all the gap equation is given by

$$1 = \lambda \int_0^{E_c} d\xi < \frac{\cos^2 2\phi}{E} (\tanh \frac{E+h}{2T} + \tanh \frac{E-h}{2T}) > \tag{1}$$

where $E = \sqrt{\xi^2 + \Delta^2 \cos^2 2\phi}$, $h = \mu_B B$ with the μ_B the Bohr magneton, and λ is the dimensionless coupling constant. Also $< \ldots >$means the average over ϕ. The above equation is transformed into

$$-\ln(\frac{\Delta}{\Delta_{00}}) = \int_0^\infty dx \Phi(x) [\frac{1}{1+e^{\beta(\Delta x + h)}} + \frac{1}{1+e^{\beta(\Delta x - h)}}] \tag{2}$$

where

$$\begin{aligned} \Phi(x) &= \frac{2}{\pi}(K(x) - E(x)) \qquad \text{for} \quad x \le 1 \\ &= \frac{2}{\pi} x (K(\frac{1}{x}) - E(\frac{1}{x})) \qquad \text{for} \quad x > 1 \end{aligned} \tag{3}$$

and Δ_{00} is the superconducting order parameter for $T = 0$ and $h = 0$, $\Delta_{00} = 2.14 T_{c0}$, T_{c0} the transition temperature at $h = 0$, and $K(x)$ and $E(x)$ are the complete elliptic integrals.

For $T,\ h << \Delta_{00}$, Eq.(2) can be expanded as

$$-\ln(\frac{\Delta}{\Delta_{00}}) \simeq 3\zeta(3)(\frac{T}{\Delta})^3 + 2\ln 2 (\frac{h}{T})^2 + \frac{1}{24}(\frac{h}{T})^4 + ... \quad \text{for} \quad \frac{h}{T} \ll 1 \tag{4}$$

$$\simeq \frac{1}{3}(\frac{h}{\Delta})^3 + \frac{\pi^2}{6}(\frac{h}{\Delta})^3 (\frac{T}{h})^2 + \ldots \qquad \text{for} \quad \frac{h}{T} \gg 1 \tag{5}$$

Eq.(2) is solved numerically and shown in Fig.1 (a) and(b). The asymptotic behavior for Δ, Eq.(5), has been mentioned already in [15]. Making use of Δ obtained in Eq.(2) the free energy of the superconducting state is given by

$$\begin{aligned} \Omega_s &= -\frac{1}{4} N_0 \Delta^2 (1 + 2 \ln \frac{\Delta_{00}}{\Delta}) - \frac{2N_0}{\beta} \int_0^{E_c} d\xi \left\langle \ln(1+e^{-\beta(E+h)}) + \log(1+e^{-\beta(E-h)}) \right\rangle \\ &= -N_0 \Delta^2 [\frac{1}{4} + \int_0^\infty dx A(x) [\frac{1}{1+e^{\beta(\Delta x + h)}} + \frac{1}{1+e^{\beta(\Delta x - h)}}]] \end{aligned} \tag{6}$$

where

$$\begin{aligned} A(x) &= \frac{2}{\pi}[(2x^2 - 1)K(x) + E(x)] \qquad \text{for} \quad x \le 1 \\ &= \frac{2}{\pi} x (K(\frac{1}{x}) + E(\frac{1}{x})) \qquad \text{for} \quad x > 1 \end{aligned} \tag{7}$$

Fig. 2. – (a) The free energy in d-wave superconducting state is shown as a function of T/T_{c0} for several fields. (b) The free energy in d-wave superconducting state is shown as a function of h/Δ_{00} for several temperatures. The dotted lines represent the free energy in the metastable state.

Fig. 3. – (a) The magnetization is shown as a function of T/T_{c0} for several fields. (b) The magnetization is shown as a function of h/Δ_{00} for several temperatures.

We show in Fig.2 (a) and (b), $\Omega_s/(N_0\Delta_{00}^2/4)$ as function of h/Δ_{00} for several T/T_{c0} and as function of T/T_{c0} for several h/Δ_{00}, respectively. For $T, h << \Delta_{00}$,

$$\Omega_s = -N_0\Delta^2(\frac{1}{4}+\frac{9}{2}\zeta(3)(\frac{T}{\Delta})^3+3\ln 2(\frac{T}{\Delta})^3(\frac{h}{T})^2+...) \qquad \text{for} \qquad \frac{h}{T}\ll 1 \tag{8}$$

$$= -N_0\Delta^2(\frac{1}{4}+\frac{1}{2}(\frac{h}{\Delta})^3+\frac{\pi^2}{2}(\frac{T}{\Delta})^3(\frac{h}{T})+O(e^{-h/T})) \qquad \text{for} \qquad \frac{h}{T}\gg 1 \tag{9}$$

Similarly the magnetization (due to spin) and the specific heat are given by

$$M = -2N_0\Delta\int_0^\infty dx\frac{N(x)}{N_0}(\frac{1}{1+e^{\beta(\Delta x+h)}}-\frac{1}{1+e^{\beta(\Delta x-h)}}) \tag{10}$$

and

$$\begin{aligned} C_s \quad = \quad & 2\beta^2 N_0\int_0^\infty dE[f(E+h)(1-f(E+h))\{(E+h)^2\frac{N(\frac{E}{\Delta})}{N_0}-\frac{1}{2}\frac{E+h}{E}T\frac{\partial\Delta^2}{\partial T}R(\frac{E}{\Delta})\} \\ & + \quad (h \quad\longrightarrow\quad -h) \quad] \end{aligned} \tag{11}$$

where $N(x)$ and $R(x)$ are given by

$$\begin{aligned} N(x)/N_0 \quad &= \frac{2}{\pi}xK(x) \qquad && \text{for} \quad x\le 1 \\ &= \frac{2}{\pi}K(\frac{1}{x}) \qquad && \text{for} \quad x>1 \end{aligned} \tag{12}$$

and

$$\begin{aligned} R(x) \quad &= \frac{2}{\pi}x[K(x)-E(x)] \qquad && \text{for} \quad x\le 1 \\ &= \frac{2}{\pi}x^2[K(\tfrac{1}{x})-E(\tfrac{1}{x})] \qquad && \text{for} \quad x>1 \end{aligned} \tag{13}$$

and $N(x)$ is the quasi-particle density of states. $M/N_0\Delta_{00}$ is shown in Fig. 3 (a) and (b), while $C_s/(\frac{\pi^2}{3})N_0T_c$ is shown in Fig. 4(a). For $T, h << \Delta_{00}$ we obtain

$$\begin{aligned} M \simeq \quad & N_0\Delta[4\ln 2(\frac{T}{\Delta})^2(\frac{h}{T})+\frac{1}{6}(\frac{T}{\Delta})^2(\frac{h}{T})^3+...] \qquad && \text{for} \quad \frac{h}{T}\ll 1 \\ \simeq \quad & N_0\Delta[(\frac{h}{\Delta})^2+\frac{\pi^2}{3}(\frac{T}{\Delta})^2+O(e^{-h/T})] \qquad && \text{for} \quad \frac{h}{T}\gg 1 \end{aligned} \tag{14}$$

and

$$\begin{aligned} C_s \quad \simeq \quad & 18\zeta(3)\frac{N_0T^2}{\Delta}\left[1+O(\frac{h}{T})^4\right] \qquad && \text{for} \quad \frac{h}{T}\ll 1 \\ \simeq \quad & \frac{2\pi^2}{3}N_0\frac{Th}{\Delta}\left[1+O(e^{-h/T})\right] \qquad && \text{for} \quad \frac{h}{T}\gg 1 \end{aligned} \tag{15}$$

Fig. 4. – (a) The specific heat is shown as a function of temperature for several fields. (b) The jump in the specific heat is shown as a function of fields. It diverges as the field approaches to 0.501 Δ_{00}.

Fig. 5. – (a) The superfluid density in the plane is shown as a function of T/T_{c0} for several fields. (b) The superfluid density in the plane is shown as a function of h/Δ_{00} for several temperatures.

The jump in the specific heat is given by

$$\Delta C_s/C_N(T_c(h)) = -\frac{16}{Re[\psi^{(2)}(\frac{1}{2}+\frac{ih}{2\pi T_c(h)})]}(1+\mathrm{Im}(\psi^{(1)}(\frac{1}{2}+\frac{ih}{2\pi T_c(h)}))\frac{h}{2\pi T_c(h)})^2 \tag{16}$$

We note that it becomes sharper as we approach $h \sim 0.501\Delta$ where the order of the transition changes to the first order [19]. It is shown in Fig. 4(b).

The in-plane and the out of plane superfluid density are given by

$$\begin{aligned}\frac{\rho_{s,\mathrm{in}}(T,h)}{\rho_{s,\mathrm{in}}(0,0)} &= 1+2\int_0^\infty d\xi < \sin^2\phi(\frac{\partial f(E+h)}{\partial E}+\frac{\partial f(E-h)}{\partial E}) > \\ &= 1-\frac{1}{T}\int_0^\infty dE\frac{N(E/\Delta)}{N_0}[\mathrm{sech}^2\frac{E+h}{2T}+\mathrm{sech}^2(\frac{E-h}{2T})]\end{aligned} \tag{17}$$

and

$$\frac{\rho_{s,\mathrm{out}}(T,h)}{\rho_{s,\mathrm{out}}(0,0)} = \frac{\Delta}{\Delta_{00}}\frac{\pi}{2} < \cos 2\phi \tanh\frac{\Delta\cos 2\phi + h}{2T} > \tag{18}$$

Here we used Ambegaokar-Baratoff formula for Josephson current [21] to calculate the out-of-plane superfluid density. At T = 0K, Eq(17) and Eq(18) reduces to

$$\frac{\rho_{s,\mathrm{in}}(0,h)}{\rho_{s,\mathrm{in}}(0,0)} = 1-\frac{2}{\pi}\frac{h}{\Delta}K(\frac{h}{\Delta}) \tag{19}$$

and

$$\frac{\rho_{s,\mathrm{out}}(0,h)}{\rho_{s,\mathrm{out}}(0,0)} = \sqrt{1-(\frac{h}{\Delta})^2} \tag{20}$$

For h,T $\ll \Delta_{00}$, we have

$$\begin{aligned}\frac{\rho_{s,\mathrm{in}}(T,h)}{\rho_{s,\mathrm{in}}(0,0)} &\simeq 1-2\ln 2(\frac{T}{\Delta})-\frac{1}{4}(\frac{T}{\Delta})(\frac{h}{T})^2+... && \text{for} \quad \frac{h}{T} << 1 \\ &\simeq 1-\frac{h}{\Delta}-2(\frac{h}{\Delta})(\frac{T}{h})e^{-\frac{h}{T}}+O(e^{-2h/T}) && \text{for} \quad \frac{h}{T} >> 1\end{aligned} \tag{21}$$

and

$$\frac{\rho_{s,\mathrm{out}}(T,h)}{\rho_{s,\mathrm{out}}(0,0)} \simeq \sqrt{1-(\frac{h}{\Delta})^2}-\frac{\pi^2}{6}(\frac{T}{\Delta})^2-\frac{\pi^2}{4}(\frac{T}{\Delta})^2(\frac{h}{\Delta})^2+O((\frac{h}{\Delta})^4,(\frac{T}{\Delta})^4) \tag{22}$$

In the last case the same expression applies for $0 \le h/T < \infty$. $\rho_{s,\mathrm{in}}(T,h)$ and $\rho_{s,\mathrm{out}}(T,h)$ are shown in Fig.5 and 6, respectively.

Finally the quasi-particle density of states in a magnetic field is shown in Fig. 7. In particular the density of states exhibits a flat portion which extend for $|E| < h$. Also the peaks at $E = \pm\Delta$ splits into double peaks at $E = \pm(\Delta \pm h)$.

Fig. 6. – (a)The c-axis superfluid density is shown as a function of T/T_{c0} for several fields. (b) The c-axis superfluid density is shown as a function of h/Δ_{00} for several temperatures.

Fig. 7. – The quasi particle density of states in the superconducting state is shown as a function of field and temperature.

In summary we have studied the paramagnetic state in d-wave superconductors, which may be realized in high-T_c cuprates and κ-$(\mathrm{ET})_2$salts in the presence of a magnetic field within the conducting plane (the a-b plane and the a-c plane, respcetively) . For simplicity we have neglected the orbital effect associated with magnetic field. However in many of above systems we believe the paramagnetic effect is predominant. In this situation the most properties in the above systems exhibit simple power laws as we have described here, which should be readily accessible experimentally. Therefore the observation of these properties described should provide another signature of d-wave superconductivity in these systems.

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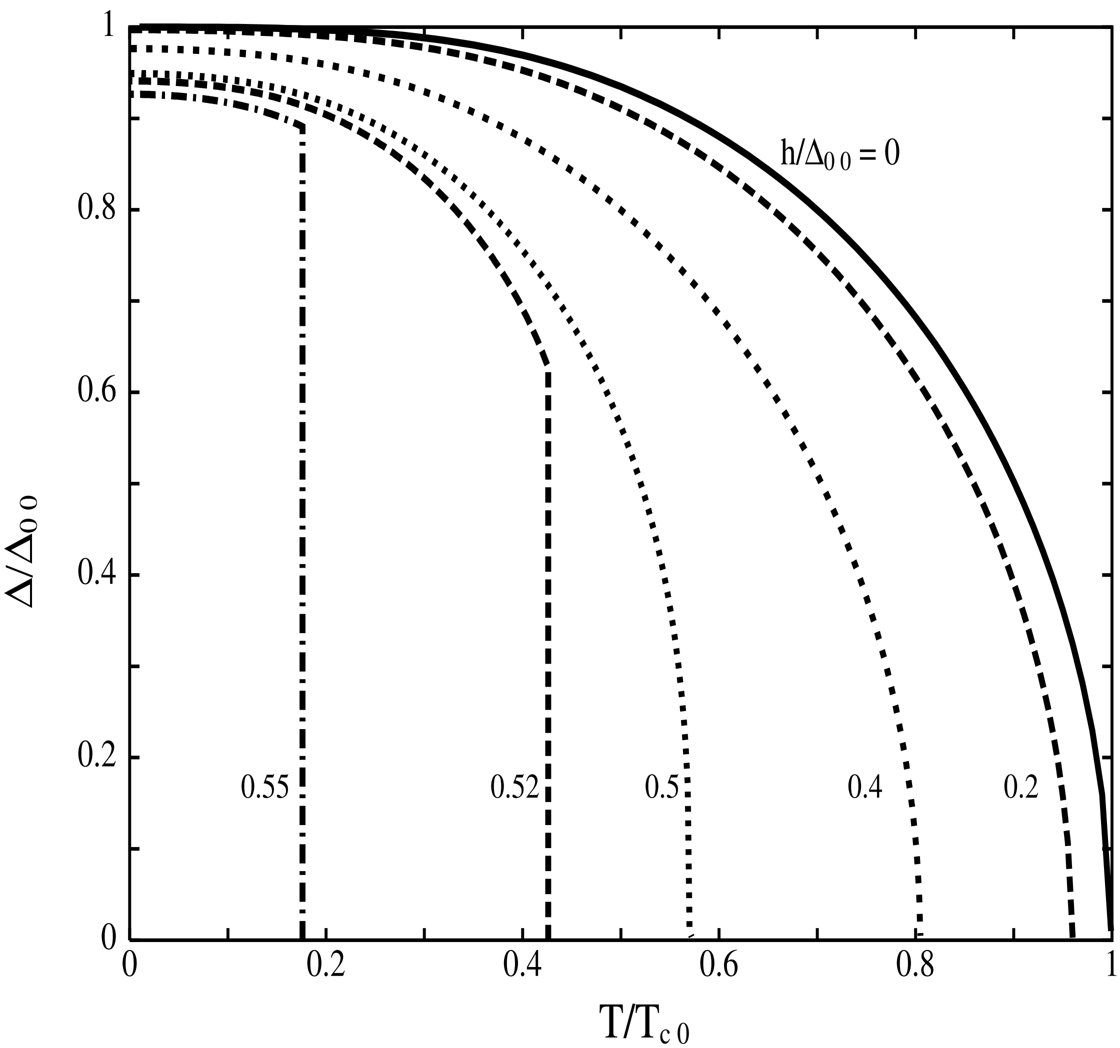
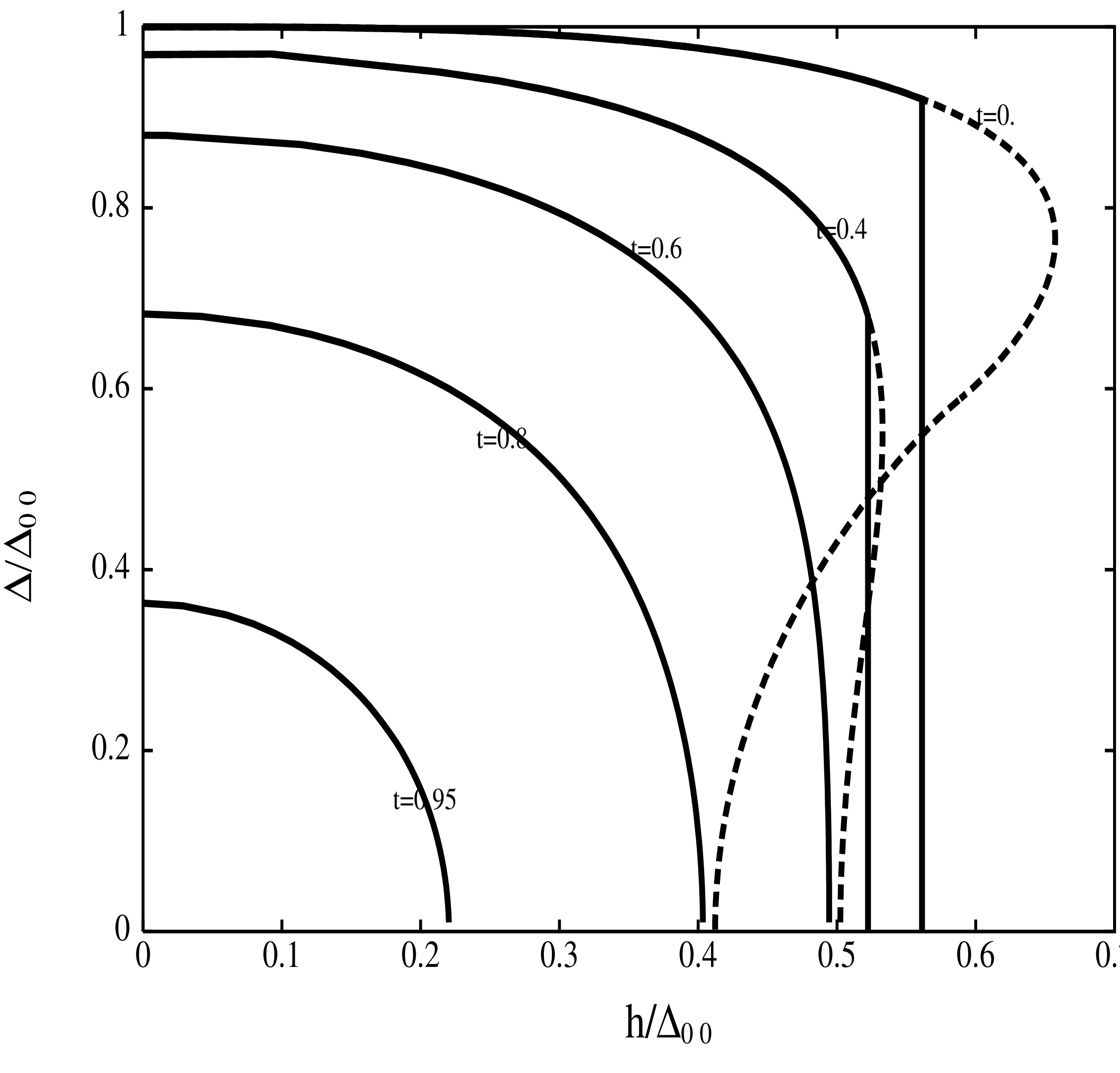


1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
Δ/Δ0 0
T/Tc 0
h/Δ0 0 = 0
0.55
0.52
0.5
0.4
0.2



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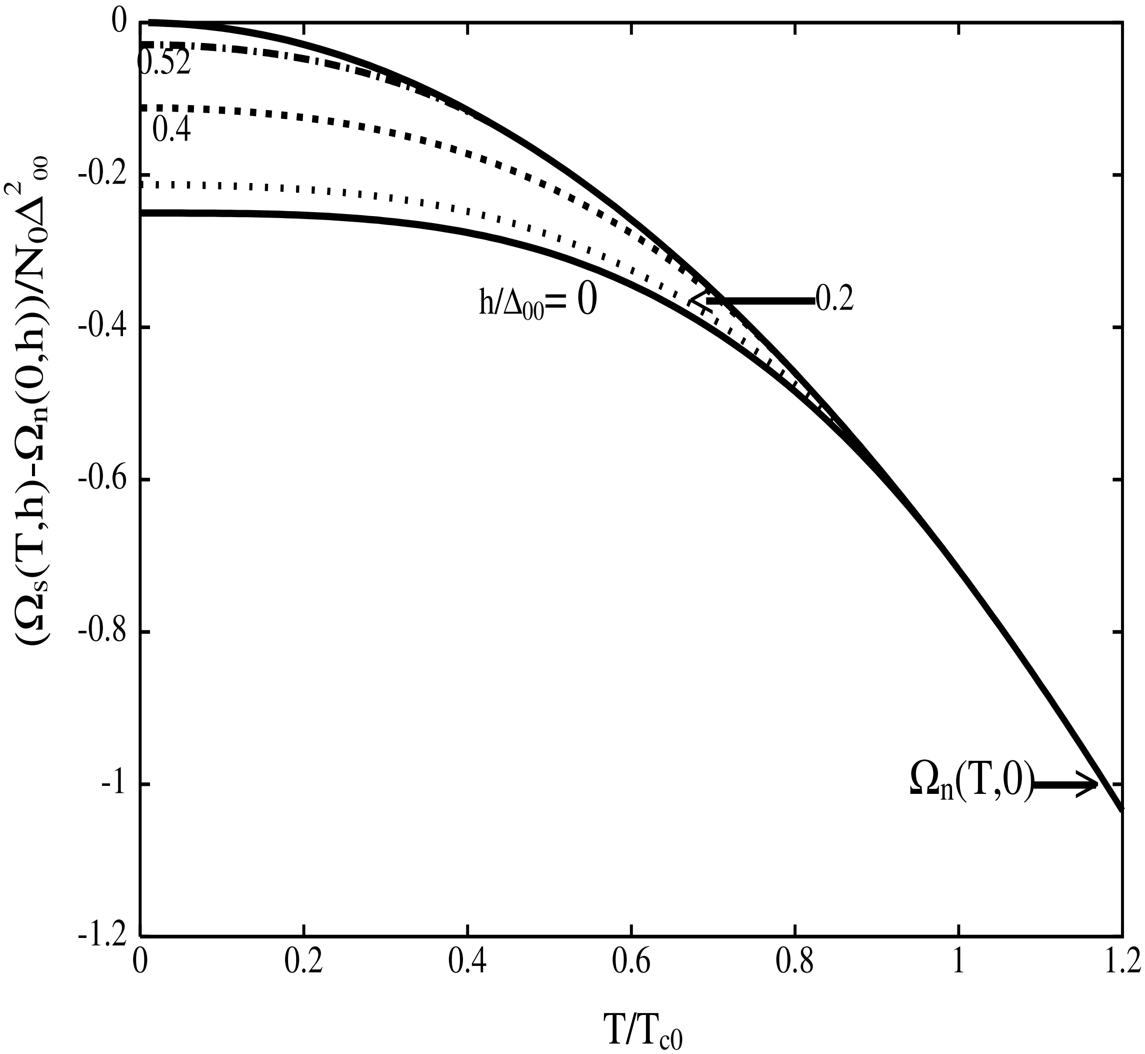
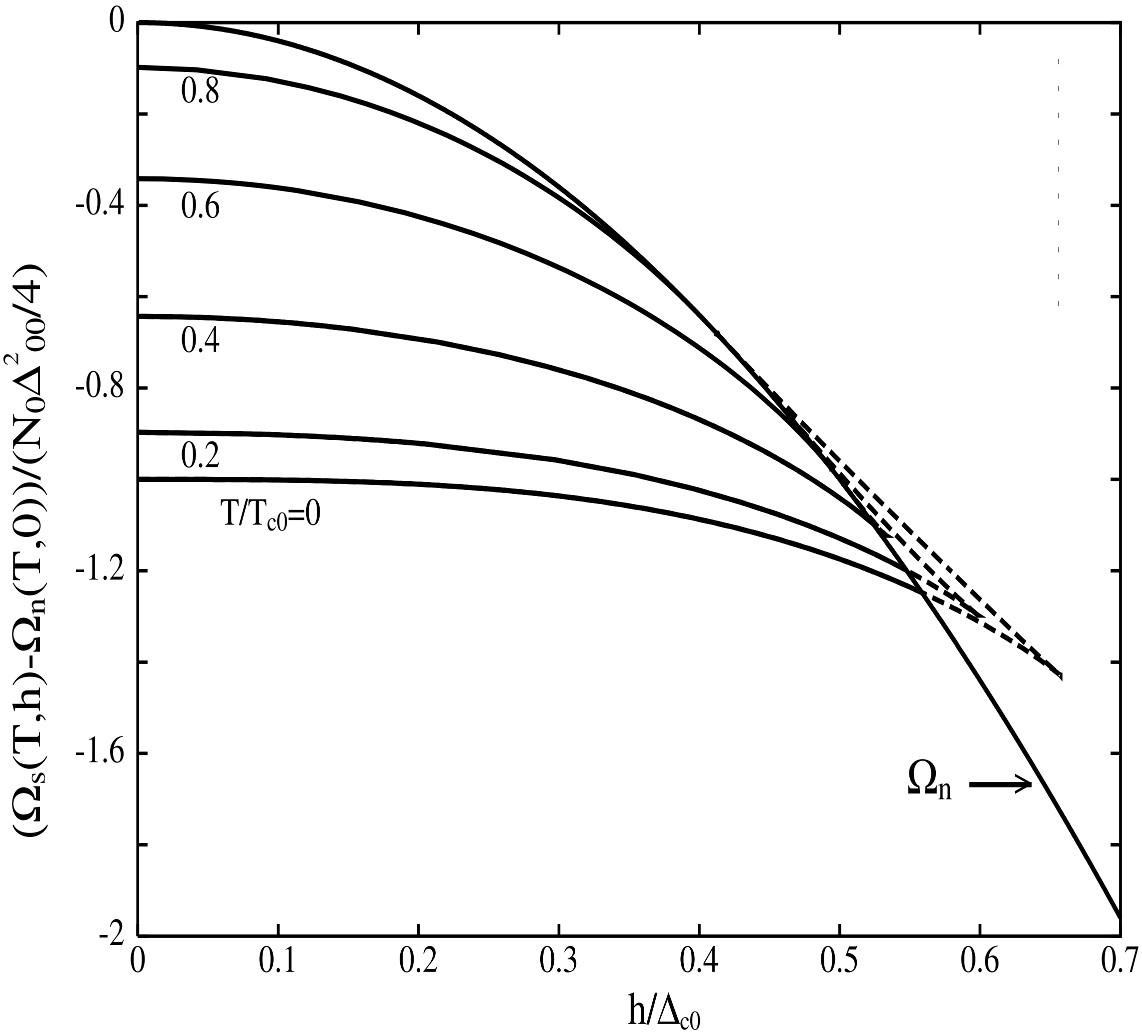
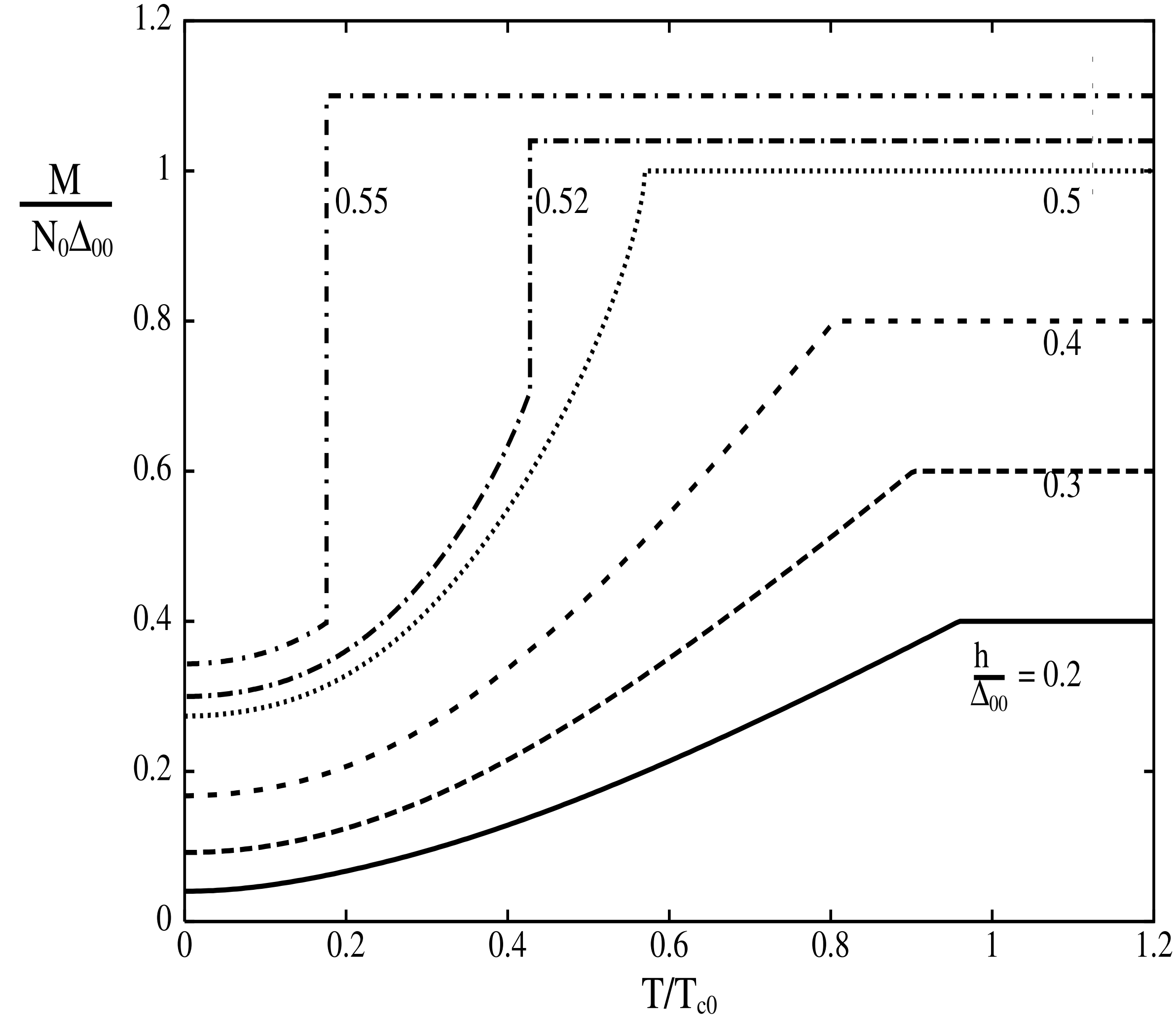
t=0.4
t=0.6
t=0.8
t=0.95
Δ/Δ0 0
h/Δ0 0
0
0.2
0.4
0.6
0.8
1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7



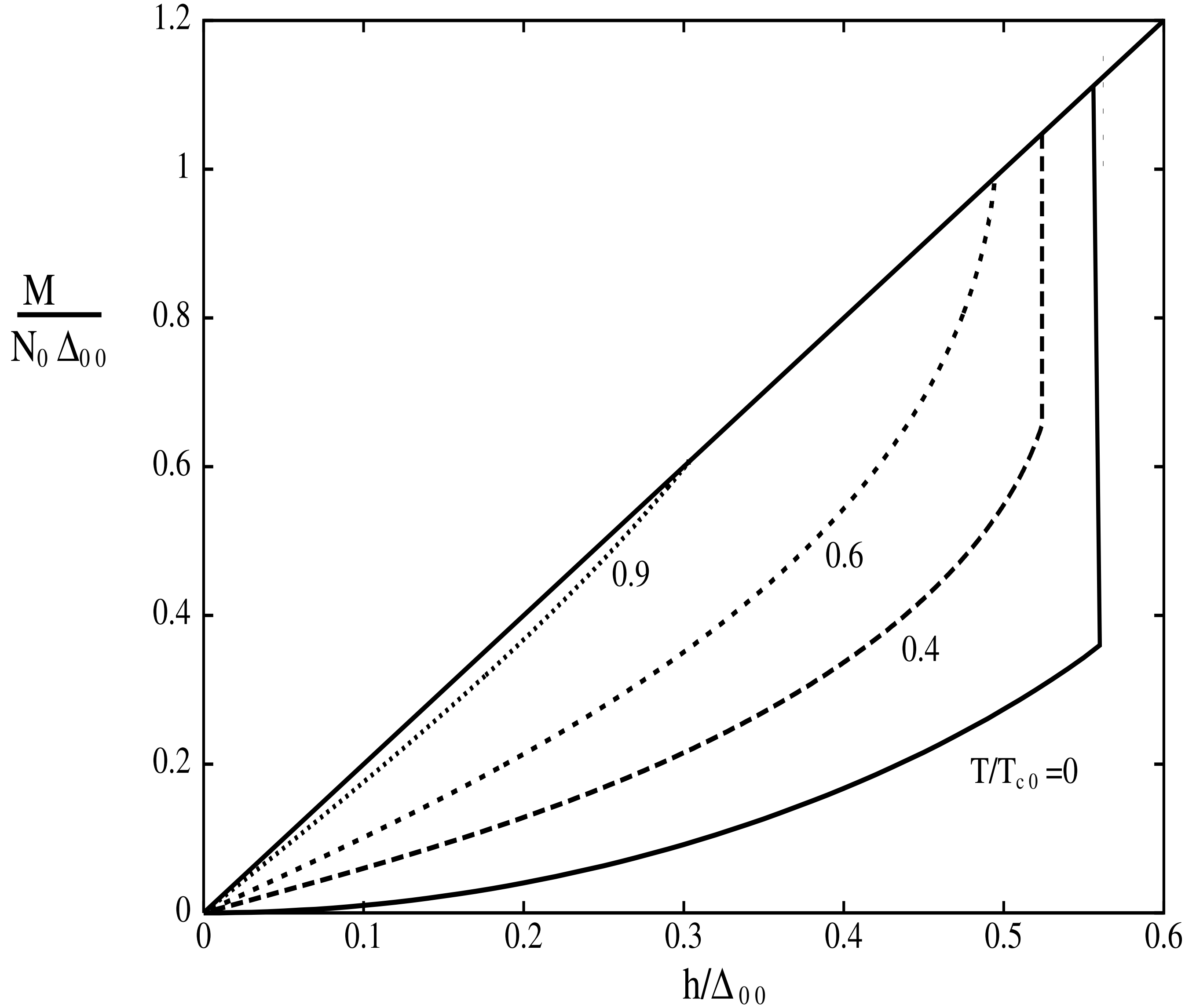
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
0
0.2
0.4
0.6
0.8
1
1.2
0.52
0.4
0.2
h/Δ00= 0
Ωn(T,0)
(Ωs(T,h)-Ωn(0,h))/N0Δ²00
T/Tc0



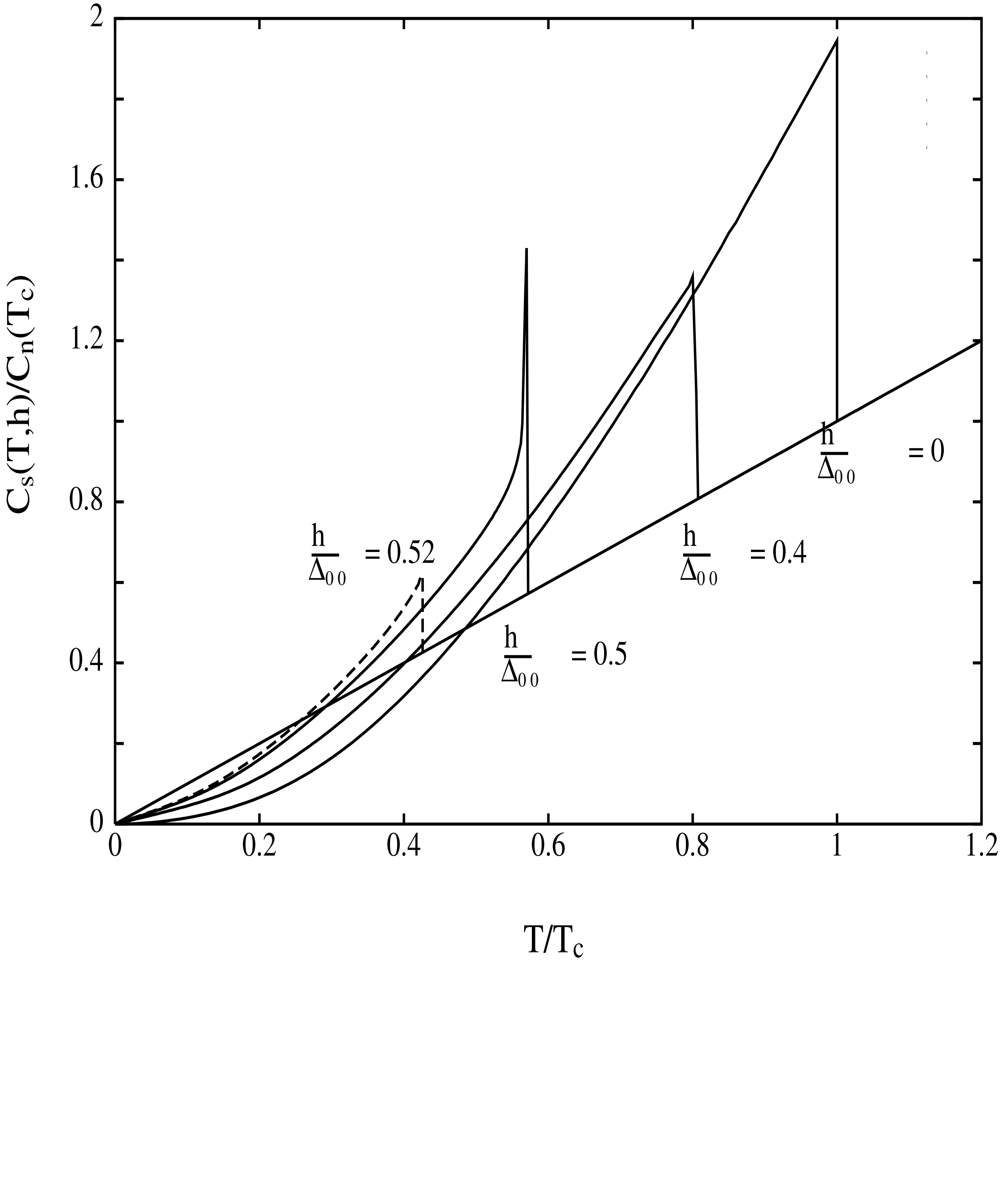
0
-0.4
-0.8
-1.2
-1.6
-2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.6
0.4
0.2
$T/T_{c0}=0$
Ω_n
$(\Omega_s(T,h)-\Omega_n(T,0))/(N_0\Delta^2_{00}/4)$
h/Δ_{c0}



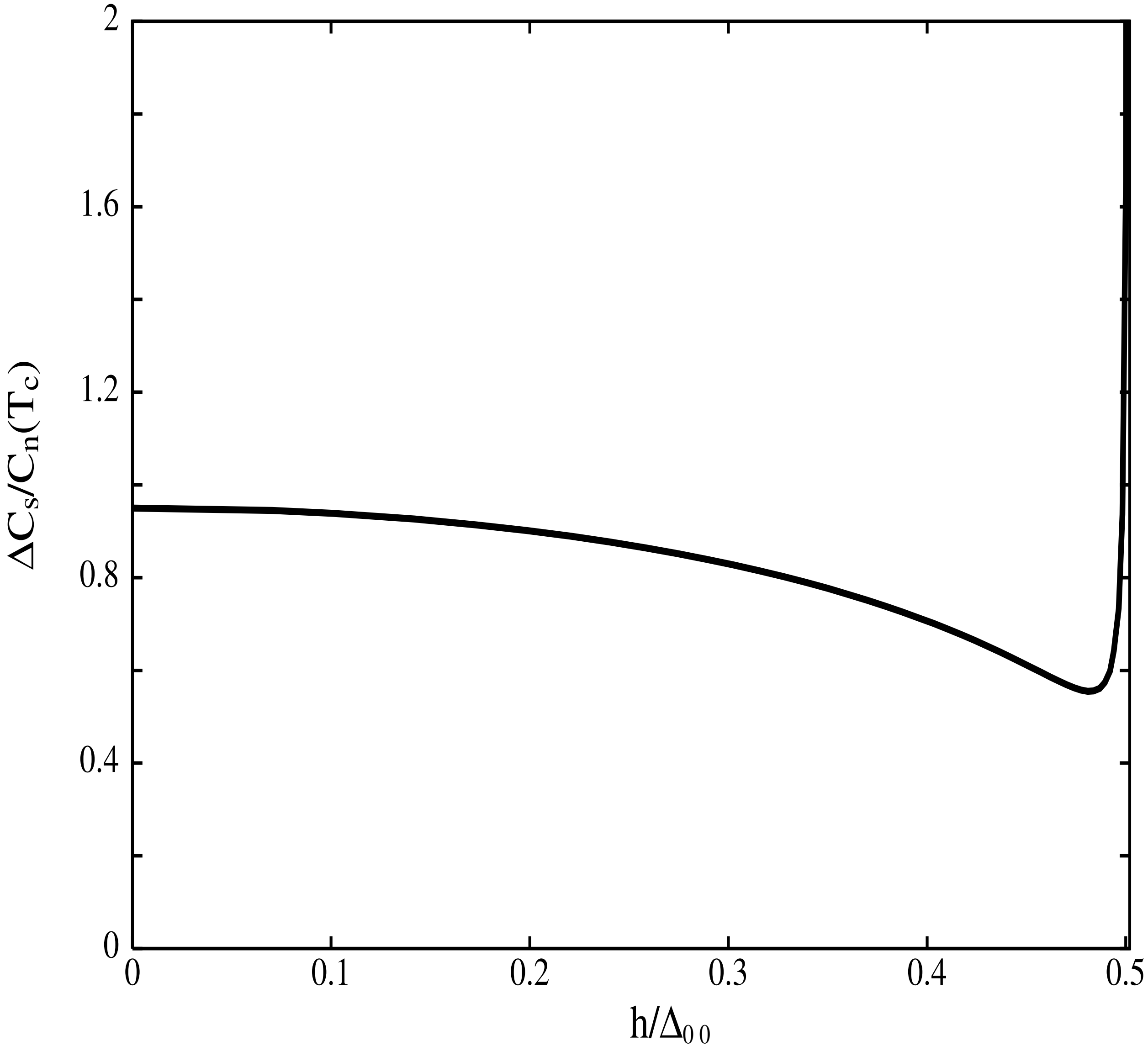
1.2
1
0.8
0.6
0.4
0.2
0
$\frac{M}{N_0\Delta_{00}}$
0
0.2
0.4
0.6
0.8
1
1.2
T/T_{c0}
0.55
0.52
0.5
0.4
0.3
$\frac{h}{\Delta_{00}} = 0.2$



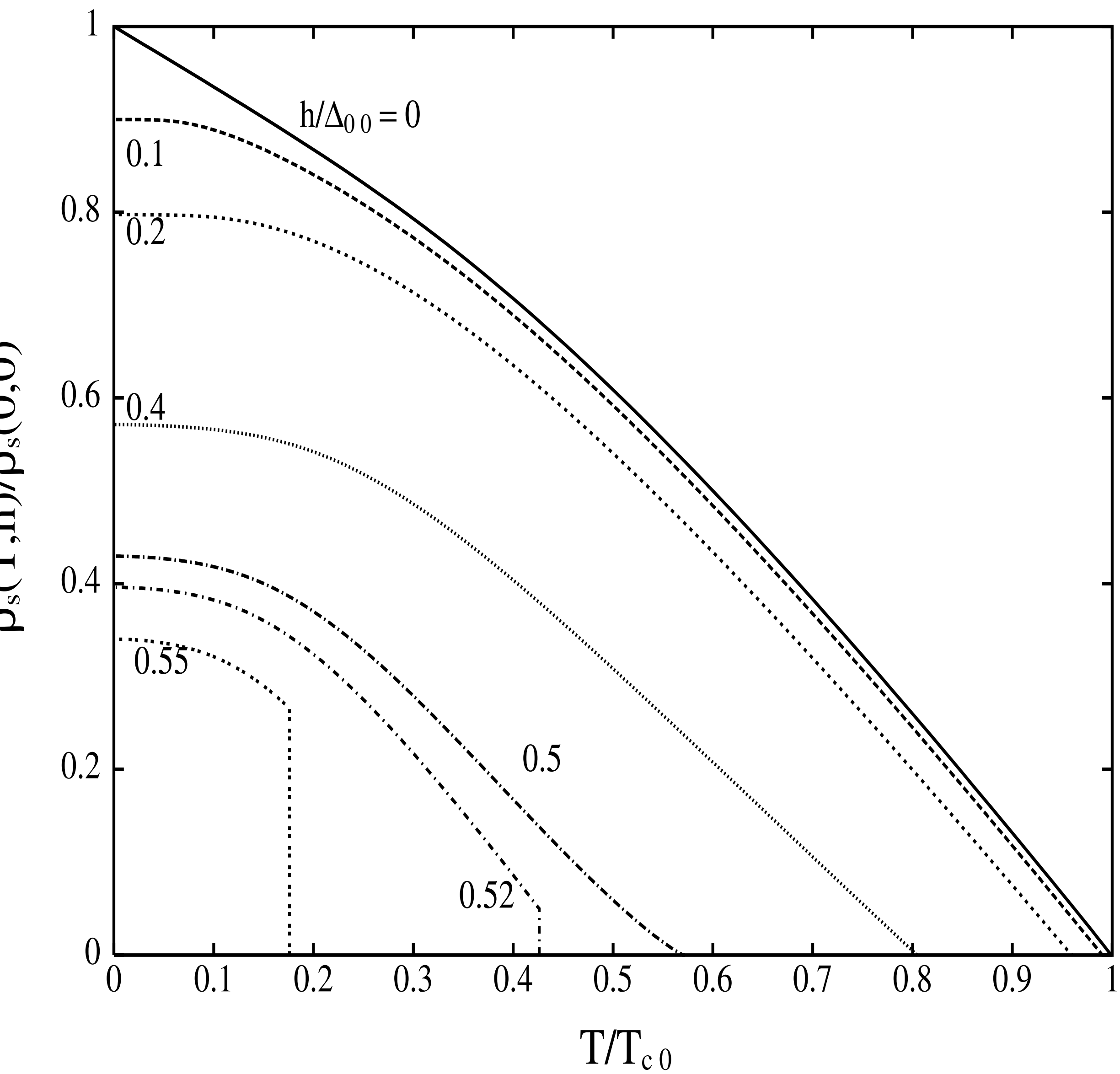
1.2
1
0.8
0.6
0.4
0.2
0
$\frac{M}{N_0 \Delta_{00}}$
0
0.1
0.2
0.3
0.4
0.5
0.6
h/Δ_{00}
0.9
0.6
0.4
$T/T_{c0} = 0$



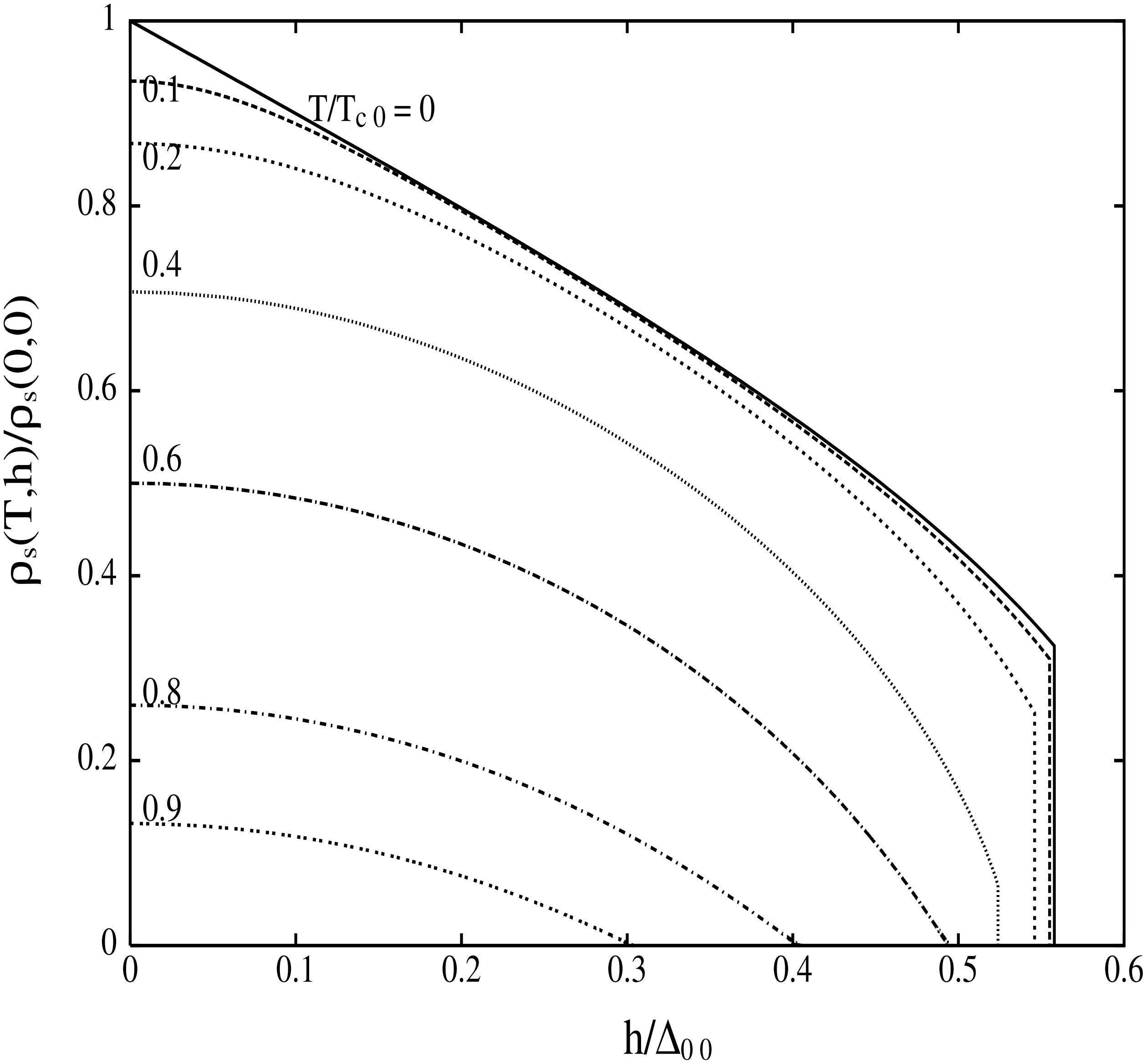
$C_s(T,h)/C_n(T_c)$
T/T_c
0
0.2
0.4
0.6
0.8
1
1.2
0.4
0.8
1.2
1.6
2
$\frac{h}{\Delta_{00}} = 0.52$
$\frac{h}{\Delta_{00}} = 0.5$
$\frac{h}{\Delta_{00}} = 0.4$
$\frac{h}{\Delta_{00}} = 0$



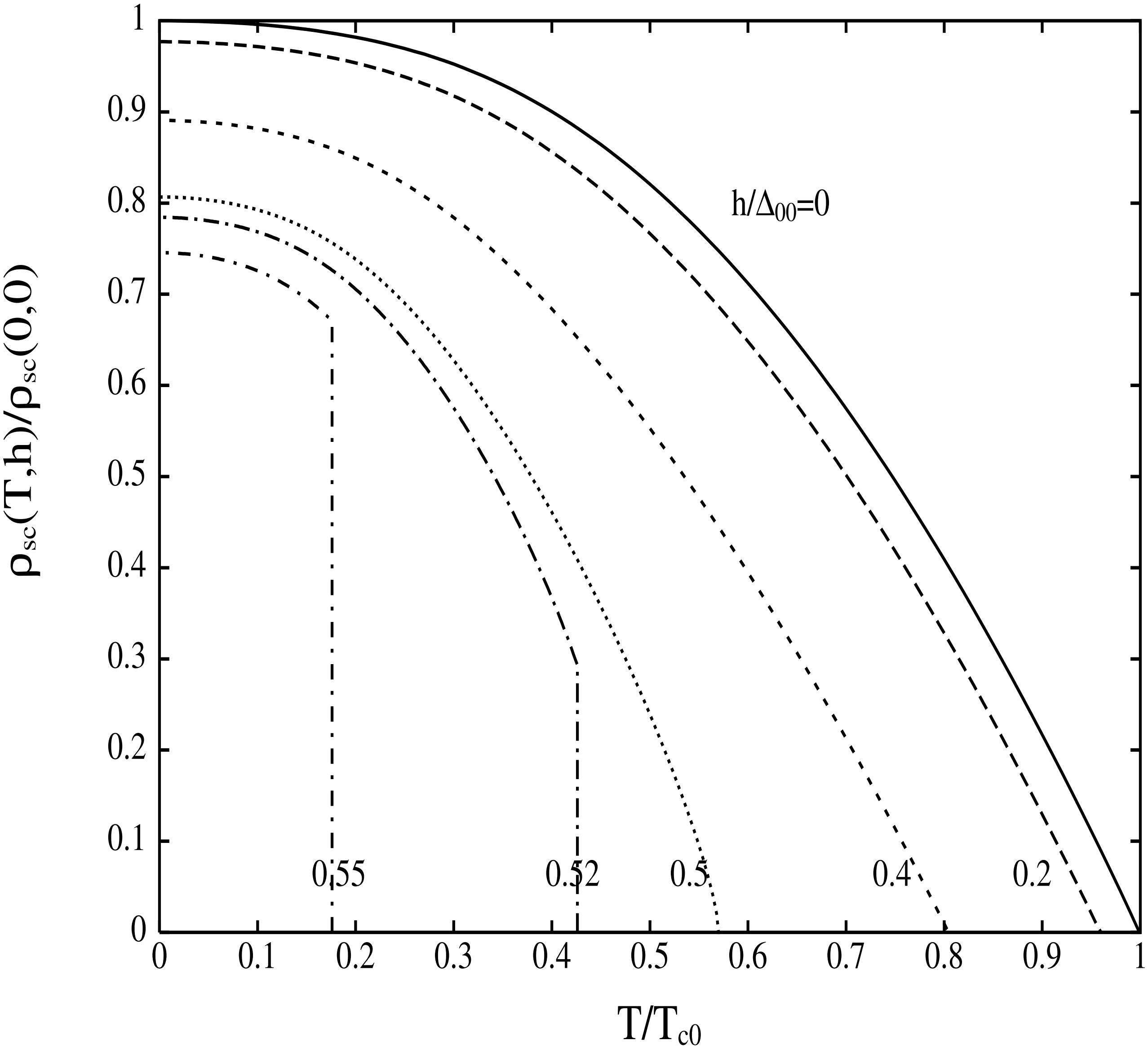
2
1.6
1.2
0.8
0.4
0
0
0.1
0.2
0.3
0.4
0.5
$\Delta C_s/C_n(T_c)$
h/Δ_{00}



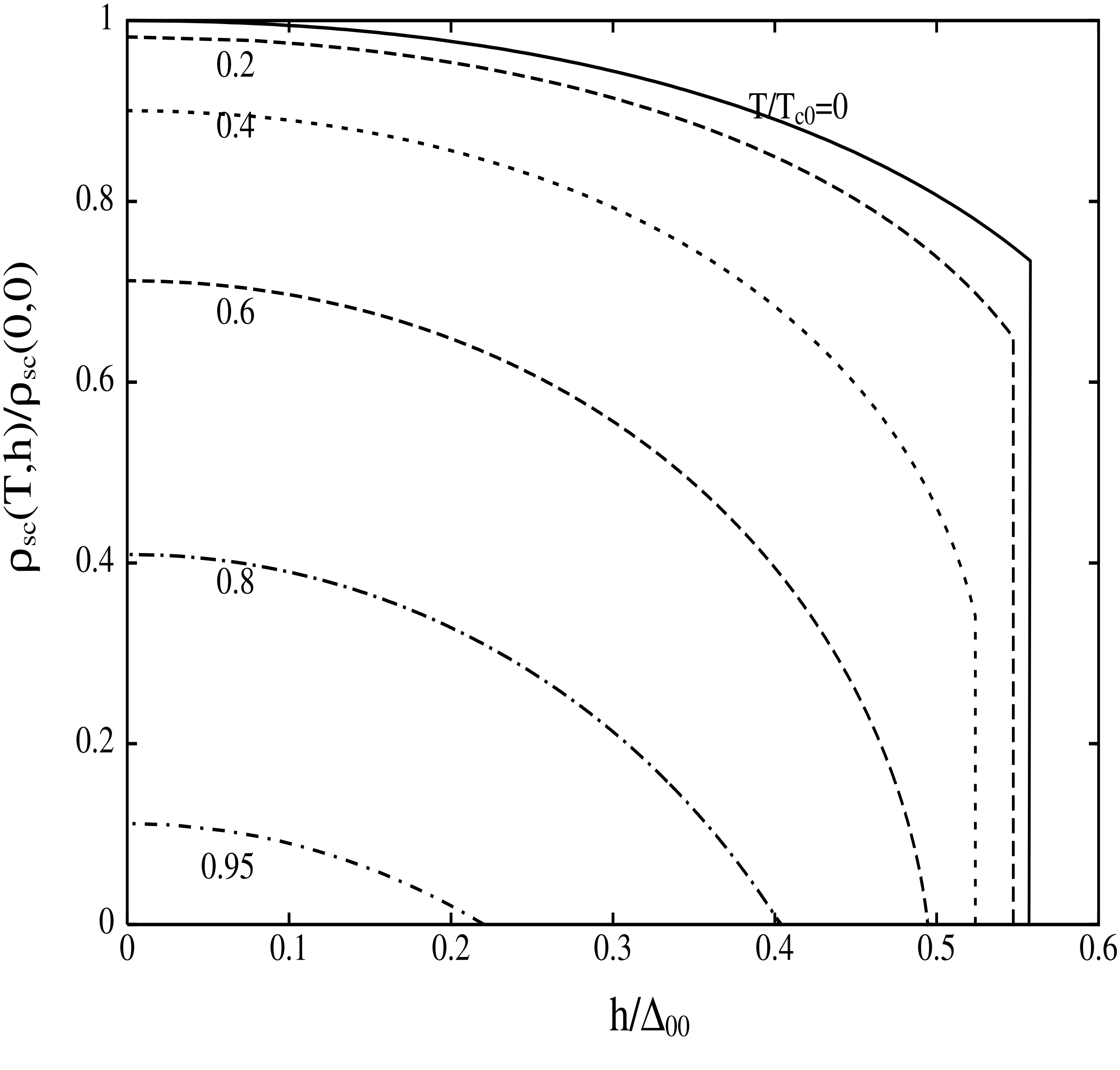
$h/\Delta_{00} = 0$
0.1
0.2
0.4
0.5
0.52
0.55
$\rho_s(T,h)/\rho_s(0,0)$
T/T_{c0}
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



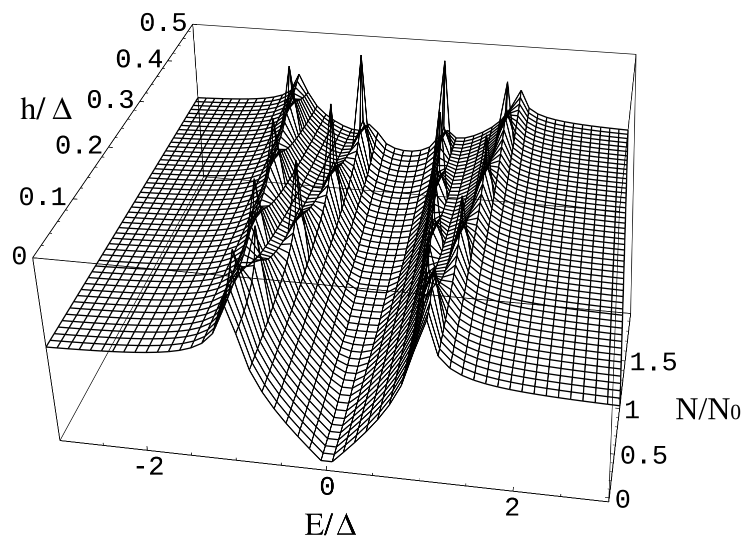
$T/T_{c\,0} = 0$
0.1
0.2
0.4
0.6
0.8
0.9
$\rho_s(T,h)/\rho_s(0,0)$
$h/\Delta_{0\,0}$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.8
1



$\rho_{sc}(T,h)/\rho_{sc}(0,0)$
T/T_{c0}
$h/\Delta_{00}=0$
0.2
0.4
0.5
0.52
0.55
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



ρsc(T,h)/ρsc(0,0)
h/Δ00
T/Tc0=0
0.2
0.4
0.6
0.8
0.95
0
0.1
0.2
0.3
0.4
0.5
0.6
0
0.2
0.4
0.6
0.8
1



0.5
0.4
0.3
0.2
0.1
0
h/ Δ
1.5
1
0.5
0
N/N0
-2
0
2
E/Δ